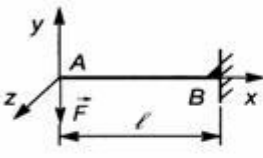
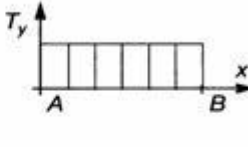
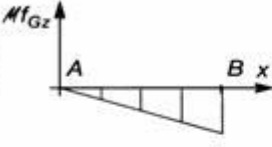
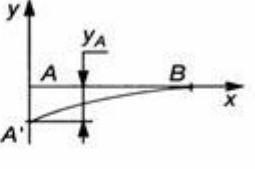
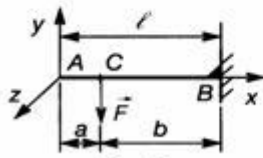
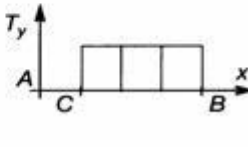
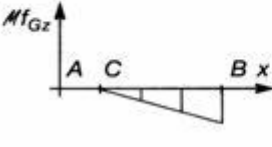
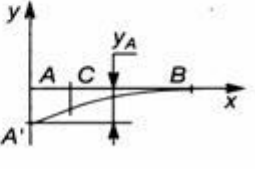
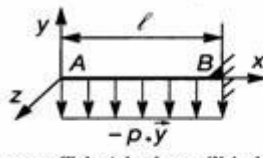
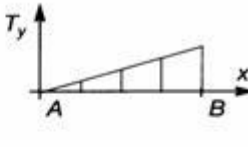
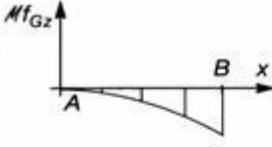
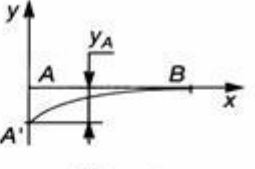
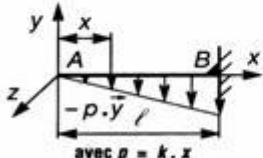
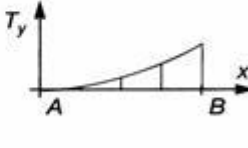

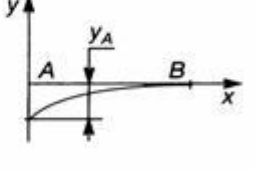
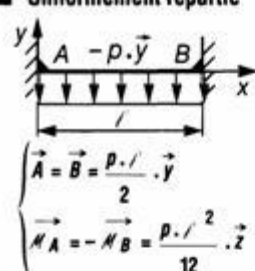
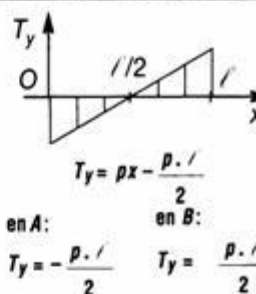
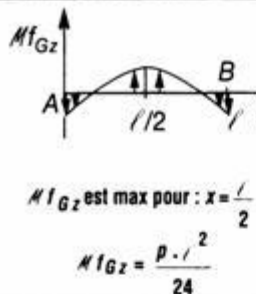
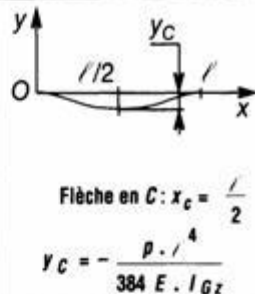
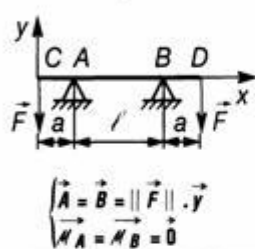
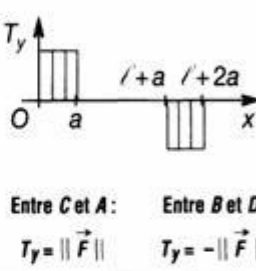
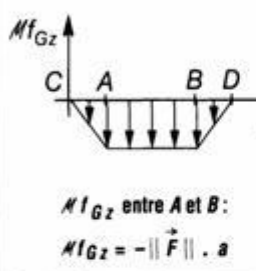
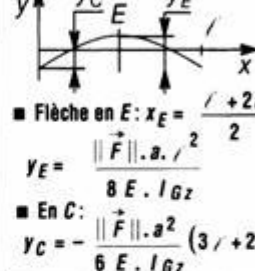
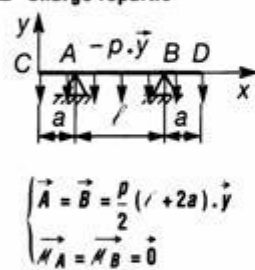
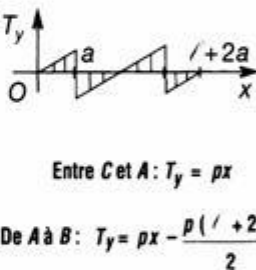
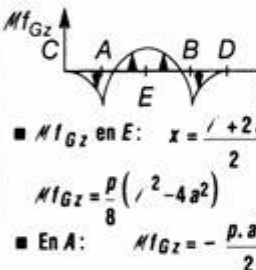
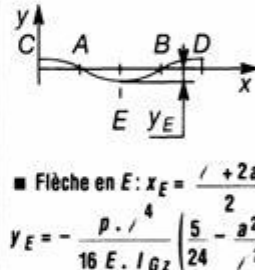
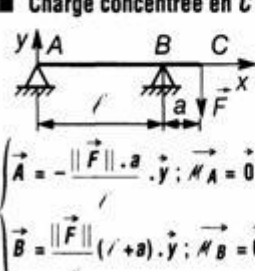
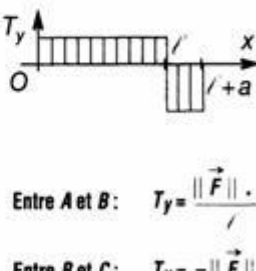
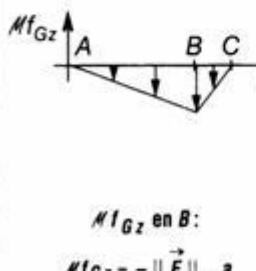
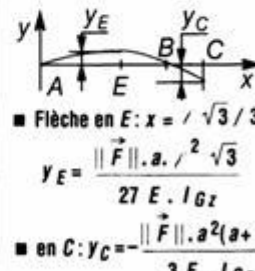
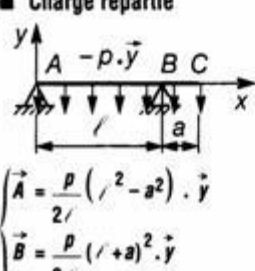
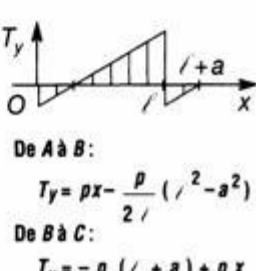
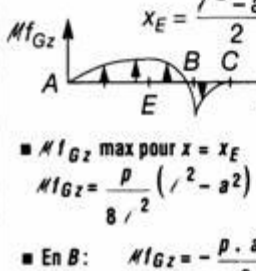
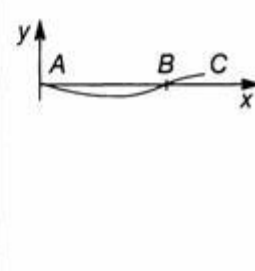
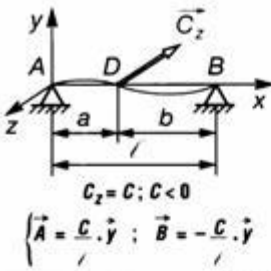
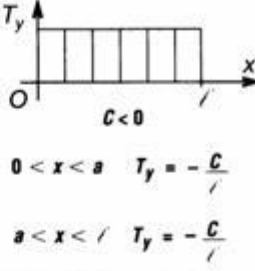
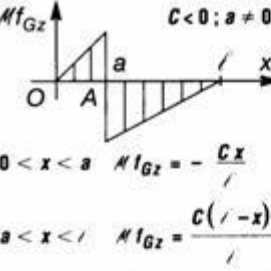
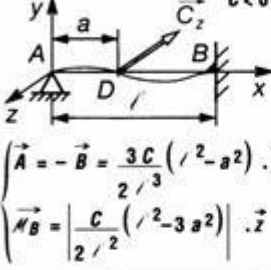
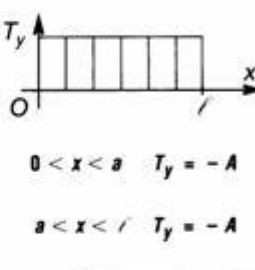
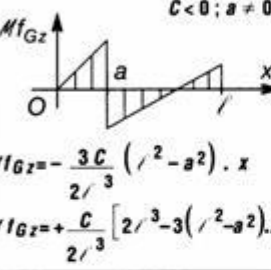
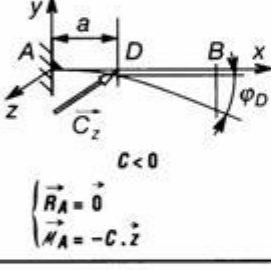
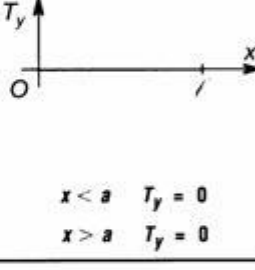
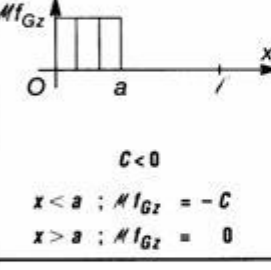
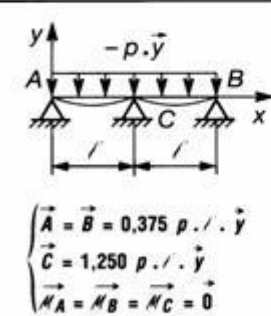
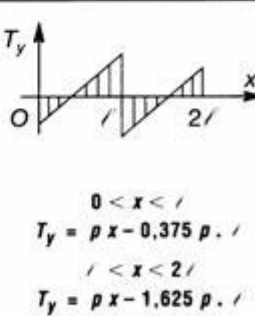
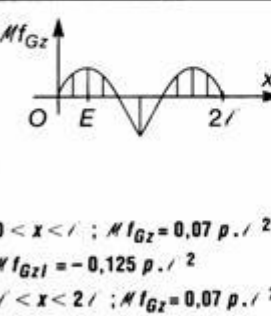
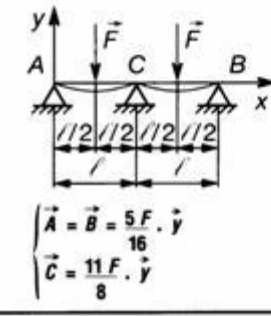

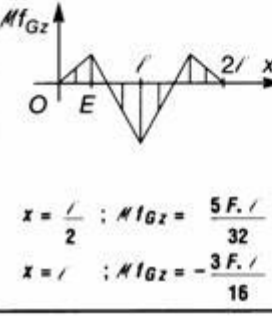


## La Flexion :

POUTRES SUR UN APPUI			
Charges - Appuis	Effort tranchant	Moment de flexion	Déformation
<p>■ Concentrée en A</p>  $\begin{cases} \vec{B} = -\vec{F} = \ \vec{F}\  \cdot \vec{y} \\ (\text{avec } F < 0) \\ \vec{M}_B = -\ \vec{F}\  \cdot l \cdot \vec{z} \end{cases}$	 <p style="text-align: center;">avec <math>F &lt; 0</math></p> <p style="text-align: center;"><math>T_y = +\ \vec{F}\ </math></p> <p style="text-align: center;">constant entre A et B</p>	 <p style="text-align: center;">avec <math>F &lt; 0</math></p> <p style="text-align: center;">Moment de flexion</p> <p style="text-align: center;">en B : <math>M_{Gz} = -\ \vec{F}\  \cdot l</math></p>	 <p style="text-align: center;">Flèche en A : <math>F &lt; 0</math></p> <p style="text-align: center;"><math>y_A = -\frac{\ \vec{F}\  \cdot l^3}{3E \cdot I_{Gz}}</math></p>
<p>■ Concentrée en C</p>  $\begin{cases} \vec{B} = -\vec{F} \\ \text{avec } F < 0 \\ \vec{B} = \ \vec{F}\  \cdot \vec{y} \\ \vec{M}_B = -\ \vec{F}\  \cdot b \cdot \vec{z} \end{cases}$	 <p style="text-align: center;">Entre A et C : <math>T_y = 0</math></p> <p style="text-align: center;">Entre C et B : avec <math>F &lt; 0</math></p> <p style="text-align: center;"><math>T_y = \ \vec{F}\ </math></p>	 <p style="text-align: center;">Moment de flexion en B : avec <math>F &lt; 0</math></p> <p style="text-align: center;"><math>M_{Gz} = -\ \vec{F}\  \cdot b</math></p>	 <p style="text-align: center;">Flèche en A :</p> <p style="text-align: center;"><math>y_A = -\frac{\ \vec{F}\  \cdot (l-a)^2 (2l+a)}{6E \cdot I_{Gz}}</math></p>
<p>■ Uniformément répartie</p>  <p><math>p</math> : coefficient de charge (N/m)</p> $\begin{cases} \vec{B} = p \cdot l \cdot \vec{y} \\ \vec{M}_B = -\frac{p \cdot l^2}{2} \cdot \vec{z} \end{cases}$	 <p style="text-align: center;">Effort tranchant max en B :</p> <p style="text-align: center;"><math>T_{y \max} = p \cdot l</math></p>	 <p style="text-align: center;">Moment de flexion en B :</p> <p style="text-align: center;"><math>M_{Gz} = -\frac{p \cdot l^2}{2}</math></p>	 <p style="text-align: center;">Flèche en A :</p> <p style="text-align: center;"><math>y_A = -\frac{p \cdot l^4}{8E \cdot I_{Gz}}</math></p>
<p>■ Linéairement répartie</p>  <p>avec <math>p = k \cdot x</math></p> $\begin{cases} \vec{B} = \frac{k \cdot l^2}{2} \cdot \vec{y} \\ \vec{M}_B = -\frac{k \cdot l^3}{6} \cdot \vec{z} \end{cases}$	 <p style="text-align: center;">Effort tranchant max en B :</p> <p style="text-align: center;"><math>T_{y \max} = \frac{k \cdot l^2}{2}</math></p>	 <p style="text-align: center;">Moment de flexion en B :</p> <p style="text-align: center;"><math>M_{Gz} = -\frac{k \cdot l^3}{6}</math></p>	 <p style="text-align: center;">Flèche en A :</p> <p style="text-align: center;"><math>y_A = -\frac{k \cdot l^5}{30E \cdot I_{Gz}}</math></p>

Charges - Appuis	Effort tranchant	Moment de flexion	Déformation
<p>■ Uniformément répartie</p>  <p> <math>\vec{A} = \vec{B} = \frac{p \cdot l}{2} \cdot \vec{y}</math>  <math>\vec{M}_A = -\vec{M}_B = \frac{p \cdot l^2}{12} \cdot \vec{z}</math> </p>	 <p> <math>T_y = px - \frac{p \cdot l}{2}</math>                      en A: <math>T_y = -\frac{p \cdot l}{2}</math>      en B: <math>T_y = \frac{p \cdot l}{2}</math> </p>	 <p> <math>Mf_{Gz}</math> est max pour : <math>x = \frac{l}{2}</math>  <math>Mf_{Gz} = \frac{p \cdot l^2}{24}</math> </p>	 <p>                     Flèche en C: <math>x_C = \frac{l}{2}</math>  <math>y_C = -\frac{p \cdot l^4}{384 E \cdot I_{Gz}}</math> </p>
POUTRE SUR DEUX APPUIS AVEC PORTE-À-FAUX SYMÉTRIQUE			
Charges - Appuis	Effort tranchant	Moment de flexion	Déformation
<p>■ Deux charges concentrées</p>  <p> <math>\vec{A} = \vec{B} = \ \vec{F}\  \cdot \vec{y}</math>  <math>\vec{M}_A = \vec{M}_B = \vec{0}</math> </p>	 <p>                     Entre C et A: <math>T_y = \ \vec{F}\ </math>      Entre B et D: <math>T_y = -\ \vec{F}\ </math> </p>	 <p> <math>Mf_{Gz}</math> entre A et B: <math>Mf_{Gz} = -\ \vec{F}\  \cdot a</math> </p>	 <p>                     Flèche en E: <math>x_E = \frac{l+2a}{2}</math>  <math>y_E = \frac{\ \vec{F}\  \cdot a \cdot l^2}{8 E \cdot I_{Gz}}</math>                      En C: <math>y_C = -\frac{\ \vec{F}\  \cdot a^2}{6 E \cdot I_{Gz}} (3l+2a)</math> </p>
<p>■ Charge répartie</p>  <p> <math>\vec{A} = \vec{B} = \frac{p \cdot (l+2a)}{2} \cdot \vec{y}</math>  <math>\vec{M}_A = \vec{M}_B = \vec{0}</math> </p>	 <p>                     Entre C et A: <math>T_y = px</math>                      De A à B: <math>T_y = px - \frac{p \cdot (l+2a)}{2}</math> </p>	 <p> <math>Mf_{Gz}</math> en E: <math>x = \frac{l+2a}{2}</math>  <math>Mf_{Gz} = \frac{p \cdot (l^2 - 4a^2)}{8}</math>                      En A: <math>Mf_{Gz} = -\frac{p \cdot a^2}{2}</math> </p>	 <p>                     Flèche en E: <math>x_E = \frac{l+2a}{2}</math>  <math>y_E = -\frac{p \cdot l^4}{16 E \cdot I_{Gz}} \left( \frac{5}{24} - \frac{a^2}{l^2} \right)</math> </p>
POUTRES SUR DEUX APPUIS AVEC PORTE-À-FAUX UNILATÉRAL			
Charges - Appuis	Effort tranchant	Moment de flexion	Déformation
<p>■ Charge concentrée en C</p>  <p> <math>\vec{A} = -\frac{\ \vec{F}\  \cdot a}{l} \cdot \vec{y}; \vec{M}_A = \vec{0}</math>  <math>\vec{B} = \frac{\ \vec{F}\  \cdot (l+a)}{l} \cdot \vec{y}; \vec{M}_B = \vec{0}</math> </p>	 <p>                     Entre A et B: <math>T_y = \frac{\ \vec{F}\  \cdot a}{l}</math>                      Entre B et C: <math>T_y = -\ \vec{F}\ </math> </p>	 <p> <math>Mf_{Gz}</math> en B: <math>Mf_{Gz} = -\ \vec{F}\  \cdot a</math> </p>	 <p>                     Flèche en E: <math>x = \frac{l \cdot \sqrt{3}}{3}</math>  <math>y_E = \frac{\ \vec{F}\  \cdot a \cdot l^2 \cdot \sqrt{3}}{27 E \cdot I_{Gz}}</math>                      en C: <math>y_C = -\frac{\ \vec{F}\  \cdot a^2 (a+l)}{3 E \cdot I_{Gz}}</math> </p>
<p>■ Charge répartie</p>  <p> <math>\vec{A} = \frac{p \cdot (l^2 - a^2)}{2l} \cdot \vec{y}</math>  <math>\vec{B} = \frac{p \cdot (l+a)^2}{2l} \cdot \vec{y}</math> </p>	 <p>                     De A à B: <math>T_y = px - \frac{p \cdot (l^2 - a^2)}{2l}</math>                      De B à C: <math>T_y = -p \cdot (l+a) + px</math> </p>	 <p> <math>Mf_{Gz}</math> max pour <math>x = x_E</math>  <math>x_E = \frac{l^2 - a^2}{2}</math>  <math>Mf_{Gz} = \frac{p \cdot (l^2 - a^2)}{8l^2}</math>                      En B: <math>Mf_{Gz} = -\frac{p \cdot a^2}{2}</math> </p>	

POUTRES SUPPORTANT UN COUPLE			
Charges - Déformées	Effort tranchant	Moment de flexion	Déformation
 <p><math>C_2 = C; C &lt; 0</math>  <math>\vec{A} = \frac{C}{l} \cdot \vec{y}; \vec{B} = -\frac{C}{l} \cdot \vec{y}</math></p>	 <p><math>C &lt; 0</math>  <math>0 &lt; x &lt; a \quad T_y = -\frac{C}{l}</math>  <math>a &lt; x &lt; l \quad T_y = -\frac{C}{l}</math></p>	 <p><math>C &lt; 0; a \neq 0</math>  <math>0 &lt; x &lt; a \quad Mf_{Gz} = -\frac{Cx}{l}</math>  <math>a &lt; x &lt; l \quad Mf_{Gz} = \frac{C(l-x)}{l}</math></p>	<p>Flèche en D:</p> $y_D = \frac{1}{E \cdot I_{Gz}} \cdot \frac{C \cdot a \cdot b (b-a)}{3}$ $\varphi_A = -\frac{C}{6E \cdot I_{Gz} \cdot l} \cdot (l^2 - 3b^2)$ $\varphi_B = -\frac{C}{6E \cdot I_{Gz} \cdot l} \cdot (l^2 - 3a^2)$
 <p><math>C &lt; 0</math>  <math>\vec{A} = -\vec{B} = \frac{3C}{2l^3} (l^2 - a^2) \cdot \vec{y}</math>  <math>\vec{M}_B = \left  \frac{C}{2l^2} (l^2 - 3a^2) \right  \cdot \vec{z}</math></p>	 <p><math>C &lt; 0; a \neq 0</math>  <math>0 &lt; x &lt; a \quad T_y = -A</math>  <math>a &lt; x &lt; l \quad T_y = -A</math></p>	 <p><math>C &lt; 0; a \neq 0</math>  <math>Mf_{Gz} = -\frac{3C}{2l^3} (l^2 - a^2) \cdot x</math>  <math>Mf_{Gz} = +\frac{C}{2l^3} [2l^3 - 3(l^2 - a^2) \cdot x]</math></p>	<p><math>0 &lt; x &lt; a</math></p> $y = -\frac{C(l-a)x}{4E \cdot I_{Gz} \cdot l^3} \dots$ $\dots \left[ \frac{l^2}{2} (3a-l) - (l+a)x^2 \right]$ $\varphi_A = -\frac{C}{4E \cdot I_{Gz} \cdot l} (l-a)(l-3a)$
 <p><math>C &lt; 0</math>  <math>\vec{R}_A = \vec{0}</math>  <math>\vec{M}_A = -C \cdot \vec{z}</math></p>	 <p><math>x &lt; a \quad T_y = 0</math>  <math>x &gt; a \quad T_y = 0</math></p>	 <p><math>C &lt; 0</math>  <math>x &lt; a; Mf_{Gz} = -C</math>  <math>x &gt; a; Mf_{Gz} = 0</math></p>	$f_D = \frac{C \cdot a^2}{2E \cdot I_{Gz}}$ $f_B = \frac{C \cdot a}{E \cdot I_{Gz}} \left( \frac{l-a}{2} \right)$ $\varphi_D = \frac{C \cdot a}{E \cdot I_{Gz}} = \varphi_B$
POUTRES SUR TROIS APPUIS DE NIVEAU			
 <p><math>\vec{A} = \vec{B} = 0,375 p \cdot l \cdot \vec{y}</math>  <math>\vec{C} = 1,250 p \cdot l \cdot \vec{y}</math>  <math>\vec{M}_A = \vec{M}_B = \vec{M}_C = \vec{0}</math></p>	 <p><math>0 &lt; x &lt; l</math>  <math>T_y = px - 0,375 p \cdot l</math>  <math>l &lt; x &lt; 2l</math>  <math>T_y = px - 1,625 p \cdot l</math></p>	 <p><math>0 &lt; x &lt; l; Mf_{Gz} = 0,07 p \cdot l^2</math>  <math>Mf_{Gz} = -0,125 p \cdot l^2</math>  <math>l &lt; x &lt; 2l; Mf_{Gz} = 0,07 p \cdot l^2</math></p>	<p>Flèche pour <math>x_E = 0,42 l</math></p> $f_E = -0,043 \frac{p \cdot l^4}{E \cdot I_{Gz}}$
 <p><math>\vec{A} = \vec{B} = \frac{5F}{16} \cdot \vec{y}</math>  <math>\vec{C} = \frac{11F}{8} \cdot \vec{y}</math></p>	 <p><math>0 &lt; x &lt; l/2 \quad T_y = -5F/16</math>  <math>l/2 &lt; x &lt; l \quad T_y = 11F/16</math>  <math>l &lt; x &lt; 3/2 \quad T_y = -11F/16</math>  <math>3/2 &lt; x &lt; 2 \quad T_y = -5F/16</math></p>	 <p><math>x = \frac{l}{2}; Mf_{Gz} = \frac{5F \cdot l}{32}</math>  <math>x = l; Mf_{Gz} = -\frac{3F \cdot l}{16}</math></p>	<p>pour <math>x_E = \frac{l \sqrt{5}}{5}</math></p> $f_E = -\frac{F \cdot l^3}{240 E \cdot I_{Gz}}$